

$\chi_{c0,2}$  decay into light meson pairs and its implication of the scalar meson structuresQiang Zhao<sup>1,2</sup>

1) Institute of High Energy Physics, Chinese Academy of Sciences, Beijing, 100049, P.R. China and

2) Department of Physics, University of Surrey, Guildford, GU2 7XH, United Kingdom

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In light of the recent data from BES collaboration for  $\chi_{c0} \rightarrow VV$ ,  $PP$  and  $SS$ , and from CLEO-c for  $\eta\eta$ ,  $\eta'\eta'$  and  $\eta\eta'$ , we present a detailed analysis of the decays of heavy quarkonia into light meson pairs such as  $\chi_{c0,2} \rightarrow VV$ ,  $PP$  and  $SS$  in a recently proposed parametrization scheme. An overall agreement with the data is achieved in  $\chi_{c0,2} \rightarrow VV$  and  $PP$ , while in  $\chi_{c0} \rightarrow SS$  we find that a possible existence of glueball- $q\bar{q}$  mixings is correlated with the OZI-rule violations, which can be further examined at CLEO-c and BESIII in  $\chi_{c0} \rightarrow SS$  measurement.

## I. INTRODUCTION

The recent systematic measurement of the  $\chi_{c0,2} \rightarrow VV$ ,  $PP$  and  $SS$  by BES [1, 2, 3, 4, 5, 6] and CLEO Collaboration [7] largely enriches the decay information about the  $\chi_{c0,2}$ . A rather unique feature for the light hadron decay of charmonia is that the transition occurs via gluon-rich processes. At charmonium mass region, vast investigations in the literature suggest that non-perturbative QCD effects are still important and sometimes can become dominant. Through the study of the hadronic decay of charmonia, one may gain some insights into the quark-gluon transition mechanisms in the interplay between non-perturbative and perturbative QCD. One is recommended to Ref. [8] for a detailed review and prospect of the relevant issues.

Different from the  $S$ -wave quarkonia, where the annihilation of the heavy quark and antiquark is a short-distance process, the pQCD calculation of the  $P$ -wave quarkonium decays encounters infrared divergences at order  $\alpha_s^3$ . For the two photon decays of  $P$ -wave charmonia various studies can be found in the literature [9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20]. The situation becomes quite complicated in the quarkonium exclusive hadronic decays, where higher order corrections are no longer a trivial task [21]. Attempts were made by Anselmino and Murgia [22] who found that quark mass corrections became significant in  $\chi_c \rightarrow VV$ . Some distinguishable features in the angular distributions of the final-state-vector-meson decays were also pinned down. More recently Braguta *et al.* [23, 24] investigated the influence of the internal quark motions on the scalar and tensor decays into two vectors in the colour-singlet approximation. Their prediction for  $\chi_{c0} \rightarrow \omega\omega$  branching ratio was in good agreement with the data, but significant discrepancies were found for  $\chi_{c2} \rightarrow \omega\omega$  compared with the data, which may be due to the model sensitivity to the choice of the meson structure functions and possible contributions from the neglected colour-octet state [14].

Different roles played by the pQCD transitions and nonperturbative mechanisms in  $\chi_{c0,2} \rightarrow \phi\phi$  were studied by Zhou, Ping, and Zou [25], who found that the pQCD calculations for  $\chi_{c2} \rightarrow \phi\phi$  could reproduce the data, while the results for  $\chi_{c0} \rightarrow \phi\phi$  were underestimated. In contrast, they showed that nonperturbative  ${}^3P_0$  quark pair creation mechanism could enhance the  $\chi_{c0} \rightarrow \phi\phi$  branching ratio, but with rather small contributions to  $\chi_{c2} \rightarrow \phi\phi$ . Their results suggest that nonperturbative mechanisms are important in  $\chi_{c0} \rightarrow \phi\phi$ , while pQCD transitions is likely dominant in  $\chi_{c2} \rightarrow \phi\phi$ .

All these still-controversial observations make the study of the exclusive decay of  $\chi_{c0,2} \rightarrow VV$ ,  $PP$ , and  $SS$  extremely interesting. Since the decay of  $\chi_{c0,2}$  into light hadrons is via the so-called singly OZI disconnected processes (SOZI), the study of  $\chi_{c0,2} \rightarrow VV$ ,  $PP$  and  $SS$  will shed light on the OZI-rule violation phenomena, which are generally driven by nonperturbative mechanisms. Nonetheless, in the isoscalar-meson-pair decay channel, the doubly OZI disconnected process (DOZI) may also contribute. The role played by the DOZI processes and their correlations with the production mechanisms of isoscalar scalar meson  $f_0$  states are an interesting issue in the study of the structure of the light scalar mesons at  $1 \sim 2$  GeV, i.e.,  $f_0(1370)$ ,  $f_0(1500)$ ,  $f_0(1710)$ , and  $f_0(1810)$ .

In this work, we shall present a systematic analysis of the exclusive decays of  $\chi_{c0,2} \rightarrow VV$ ,  $PP$  and  $SS$  based on an improved parametrization scheme proposed recently [26]. In light of the new data from BES [6] and CLEO-c Collaboration [7], we shall identify the role played by the DOZI processes, and gain some insights into the scalar structures in  $\chi_{c0,2} \rightarrow SS$ .

The content is organized as follows: In Section II, the parametrization scheme for  $\chi_{c0,2} \rightarrow MM$  is summarized. In Section III, we present the analysis and numerical results for  $\chi_{c0,2} \rightarrow MM$  in line with the most recent data from BES and CLEO-c. A short summary will be given in Section IV.

## II. PARAMETRIZATION FOR $\chi_{c0,2} \rightarrow MM$

In Ref. [26] the decay of  $\chi_{c0,2} \rightarrow VV$ ,  $PP$  and  $SS$  was investigated in a parametrization scheme where the production of the final state hadrons were described by a set of transition amplitudes for either SOZI or DOZI processes. Such a parametrization as a leading order approximation is useful for identifying the roles played by different transition mechanisms and will avoid difficulties arising from our poor knowledge about the nonperturbative dynamics. Associated with the up-to-date experimental data, we can constrain the model parameters and make predictions which can be tested in future measurements.

The detailed definition of the parametrization was given in Ref. [26], we only summarize the main ingredients here with slightly rephrased expressions:

i) The basic transition amplitude is defined to be the  $c\bar{c}$  annihilation into two gluons which then couple to two non-strange quark pairs to form final state mesons:

$$\langle (q_1\bar{q}_2)_{M1}(q_3\bar{q}_4)_{M2}|V_0|\chi_c\rangle \equiv g_{\langle 14 \rangle}g_{\langle 23 \rangle} \equiv g_0^2 , \quad (1)$$

where  $V_0$  is the interaction potential, and  $q(\bar{q})$  is non-strange quark (antiquark) with  $g_{\langle 14 \rangle} = g_{\langle 23 \rangle} = g_0$ . Basically, such a coupling will depend on the quantum numbers of the initial quarkonium. We separate the partial decay information by introducing a conventional form factor in the calculation, i.e.,  $\mathcal{F}(|\mathbf{p}|) \equiv |\mathbf{p}|^{2l} \exp(-|\mathbf{p}|^2/8\beta^2)$  with  $\beta = 0.5$  GeV, for the relative  $l$ -wave two-body decay.

ii) To include the SU(3) flavour symmetry breaking effects, we introduce

$$R \equiv \langle (q\bar{s})_{M1}(s\bar{q})_{M2}|V_0|\chi_c\rangle/g_0^2 = \langle (s\bar{q})_{M1}(q\bar{s})_{M2}|V_0|\chi_c\rangle/g_0^2 , \quad (2)$$

which implies the occurrence of the SU(3) flavour symmetry breaking at each vertex where a pair of  $s\bar{s}$  is produced, and  $R = 1$  is in the SU(3) flavour symmetry limit. For the production of two  $s\bar{s}$  pairs via the SOZI potential, the recognition of the SU(3) flavor symmetry breaking in the transition is accordingly

$$R^2 = \langle (s\bar{s})_{M1}(s\bar{s})_{M2}|V_0|\chi_c\rangle/g_0^2 . \quad (3)$$

iii) The DOZI process is parametrized by introducing parameter  $r$  accounting for its relative strength to the SOZI amplitude:

$$r \equiv \langle (s\bar{s})_{M1}(q\bar{q})_{M2}|V_1|\chi_c\rangle/g_0^2 = \langle (q\bar{q})_{M1}(s\bar{s})_{M2}|V_1|\chi_c\rangle/g_0^2 , \quad (4)$$

where  $V_1$  denotes the interaction potential.

iv) Scalar glueball state can be produced in company with an isoscalar  $q\bar{q}$  or in pair in the final state. We parametrize their amplitudes by introducing an additional quantity  $t$  for the relative strength of the process of glueball production recoiling a  $q\bar{q}$  to the basic amplitude  $g_0^2$ :

$$\langle (q\bar{q})G|V_2|\chi_c\rangle \equiv t\langle (q\bar{q})_{M1}(q\bar{q})_{M2}|V_0|\chi_c\rangle = tg_0^2 . \quad (5)$$

A reasonable assumption for the glueball coupling is that the glueball does not pay a price to couple to  $gg$ , namely, the so-called “flavor-blind assumption” following the gluon counting rule. Under such a condition, parameter  $t$  has a value of unity, and the glueball production amplitude is of the same strength as the basic amplitude  $g_0^2$ . Similarly, the production of a glueball pair can be expressed as

$$\langle GG|V_3|\chi_c\rangle = t\langle (q\bar{q})G|V_2|\chi_c\rangle = t^2g_0^2 . \quad (6)$$

Considering a general expression for isoscalar meson pair production with  $q\bar{q}$  and glueball components, e.g.  $M_{1,2} = x_{1,2}|G\rangle + y_{1,2}|s\bar{s}\rangle + z_{1,2}|n\bar{n}\rangle$ , we can write the transition amplitude for  $\chi_c \rightarrow M_1 M_2$  as

$$\begin{aligned} & \langle M_1(I=0)M_2(I=0)|(V_0 + V_1 + V_2 + V_3)|\chi_c\rangle \\ &= \langle (x_1G + y_1s\bar{s} + z_1n\bar{n})(x_2G + y_2s\bar{s} + z_2n\bar{n})|(V_0 + V_1 + V_2 + V_3)|\chi_c\rangle \\ &= g_0^2[(x_1t(tx_2 + Ry_2 + \sqrt{2}z_2) + y_1R(tx_2 + (1+r)Ry_2 + \sqrt{2}rz_2) \\ &\quad + z_1(\sqrt{2}tx_2 + \sqrt{2}rRy_2 + (1+2r)z_2)] . \end{aligned} \quad (7)$$

For meson pair production with isospin  $I = 1/2$  and 1, the transitions only occur via potential  $V_0$ , and they can be expressed as

$$\langle M_1(I=1/2)M_2(I=1/2)|V_0|\chi_c\rangle = Rg_0^2 , \quad (8)$$

$$\langle M_1(I=1)M_2(I=1)|V_0|\chi_c\rangle = g_0^2 . \quad (9)$$

The modification of the above parametrization rule compared to Ref. [26] is on the glueball production. Here, parameters  $r$  and  $t$  are explicitly separated out. Parameter  $r$  describes the property of the  $q\bar{q}$ -gg couplings in the DOZI processes. Apparent contributions from the DOZI processes generally demonstrate the importance of the OZI-rule violations due to long-range interactions [27]. In contrast, parameter  $t$  distinguishes the  $G$ -gg coupling from the  $q\bar{q}$ -gg, and will allow us to investigate the role played by glueball productions. In the present scheme the underlying physics denoted by the parameters can be more clearly identified.

### III. DECAY OF $\chi_{c0,2} \rightarrow MM$

In this Section we revisit  $\chi_{c0,2} \rightarrow VV$ ,  $PP$  and  $SS$  taking into account the new data from both BES and CLEO-c.

#### A. $\chi_{c0,2} \rightarrow VV$

For  $\chi_{c0,2} \rightarrow VV$ , three channels, i.e.  $\phi\phi$ ,  $\omega\omega$  and  $K^{*0}\bar{K}^{*0}$ , have been measured by BES collaboration [1, 2, 3]. Since we neglect glueball component in  $\omega$  and  $\phi$ , and assume that  $\omega$  is pure  $n\bar{n}$  and  $\phi$  is pure  $s\bar{s}$  due to ideal mixing, we can determine parameters  $g_0$ ,  $r$ , and  $R$ . Predictions for  $\chi_{c0,2} \rightarrow \rho\rho$  and  $\omega\phi$  can then be made.

In Table I, the parameters are presented. In Table II, we list the fitting results for  $\chi_{c0,2} \rightarrow VV$  in comparison with the experimental data [1, 2, 3]. Also, the result by fitting the PDG average values for  $\chi_{c0,2} \rightarrow \phi\phi$ ,  $\omega\omega$  and  $K^{*0}\bar{K}^{*0}$  are included.

One apparent feature is that the OZI-rule violation and SU(3) flavor symmetry breaking are much obvious in  $\chi_{c0} \rightarrow VV$  than in  $\chi_{c2} \rightarrow VV$ . Parameter  $r$  is found to be about 20% for  $\chi_{c0}$ , while its central values are about 1% for  $\chi_{c2}$  though the uncertainties are about 10%. The consequence of small DOZI process contributions is that the production branching ratios for  $\chi_{c0,2} \rightarrow \omega\phi$  become rather small. For instance, predictions for the branching ratio of  $\chi_{c0} \rightarrow \omega\phi$  are at least one order of magnitude smaller than  $\phi\phi$  channel, and the PDG averaged values for the experimental data lead to a negligibly small branching ratio for  $\chi_{c2} \rightarrow \omega\phi$ . Further experimental measurement confirmation of this prediction will be extremely interesting.

The  $\rho\rho$  branching ratio turns to be sensitive to the experimental uncertainties carried by those available data. Different from other decay channels, which are determined by parameters  $r$ ,  $R$  and  $g_0$  in a correlated way, it only depends on parameter  $g_0$ . Therefore, the  $\rho\rho$  channel is ideal for testing this parametrization scheme, and can put further constraint on the parameters.

#### B. $\chi_{c0,2} \rightarrow PP$

Decay channels of  $\chi_{c0,2} \rightarrow \eta\eta$ ,  $K^+K^-$ ,  $K_s^0\bar{K}_s^0$  and  $\pi\pi$  have been measured at BES [1, 4, 5, 6]. However, as studied in Ref. [26], the relatively large uncertainties with  $\chi_{c0} \rightarrow \eta\eta$  brought significant errors to parameter  $r$ , and the role played by the DOZI processes cannot be clarified. It was shown in Ref. [26] that within the uncertainties of  $BR_{\chi_{c0} \rightarrow \eta\eta} = (2.1 \pm 1.1) \times 10^{-3}$  [6], the relative branching ratios of  $\chi_{c0,2} \rightarrow \eta\eta$ ,  $\eta\eta'$  and  $\eta'\eta'$  were very sensitive to the OZI-rule violation effects, and the branching ratio fractions can vary drastically. The world averaged data for  $\chi_{c0} \rightarrow K^+K^-$ ,  $K_s^0\bar{K}_s^0$ , and  $\pi\pi$  [28] do not deviate significantly from the BES data [1, 4, 5, 6] except that  $BR_{\chi_{c0} \rightarrow \eta\eta} = (1.9 \pm 0.5) \times 10^{-3}$  has much smaller errors. Recently, CLEO-c publishes their results for  $\chi_{c0,2} \rightarrow \eta\eta$ ,  $\eta'\eta'$  and  $\eta\eta'$  [7], with  $BR_{\chi_{c0} \rightarrow \eta\eta} = (3.1 \pm 0.5 \pm 0.4 \pm 0.2) \times 10^{-3}$ ,  $BR_{\chi_{c0} \rightarrow \eta\eta'} = (1.7 \pm 0.4 \pm 0.2 \pm 0.1) \times 10^{-3}$  and  $BR_{\chi_{c0} \rightarrow \eta\eta'} < 0.5 \times 10^{-3}$ . Upper limits are given for  $\chi_{c2}$ , i.e.  $BR_{\chi_{c2} \rightarrow \eta\eta} < 0.47 \times 10^{-3}$ ,  $BR_{\chi_{c2} \rightarrow \eta\eta'} < 0.31 \times 10^{-3}$ , and  $BR_{\chi_{c2} \rightarrow \eta\eta'} < 0.23 \times 10^{-3}$ .

Adopting the world-average data from PDG [28] and including the new data from CLEO-c [7], we can now make a constraint on the model parameters for  $\chi_{c0} \rightarrow PP$ . We also make a fit for  $\chi_{c2} \rightarrow PP$  in a similar way with the experimental bound limits. The fitted parameters and branching ratios are listed in Table III and IV, respectively.

It shows that the decay of  $\chi_{c0} \rightarrow PP$  can be described consistently with small  $\chi^2$ . A prominent feature is that the SU(3) flavor symmetry breaking effects turn out to be small, i.e.  $R = 1.035 \pm 0.067$  does not deviate significantly from unity. Meanwhile, parameter  $r = -0.120 \pm 0.044$  suggests that contributions from the DOZI processes are not important. The production of  $\eta\eta'$  is thus strongly suppressed which is consistent with CLEO-c results [7]. These features indicate that pQCD transitions play a dominant role in  $PP$  decay channels.

In  $\chi_{c2} \rightarrow PP$ , by fitting the PDG data and adopting the CLEO-c bound limits for  $\eta\eta$ ,  $\eta'\eta'$  and  $\eta\eta'$ , we obtain results with large  $\chi^2$ . Contrary to  $\chi_{c0} \rightarrow PP$ , the fitted parameter  $R = 0.778 \pm 0.067$  indicates significant SU(3) flavor symmetry breakings. The OZI-rule violation parameter  $r = -0.216 \pm 0.102$  also suggests that the DOZI processes are relatively more influential than in  $\chi_{c0}$ . However, this could be due to the poor status of the data. Notice that

$BR_{\chi_{c2} \rightarrow K^+ K^-} = (0.77 \pm 0.14) \times 10^{-3}$  and  $BR_{\chi_{c2} \rightarrow K_s^0 \bar{K}_s^0} = (0.67 \pm 0.11) \times 10^{-3}$  have violated the isospin relation drastically. It needs further experiment to check whether this is due to datum inconsistency or unknown mechanisms.

It is interesting to see the change of the branching ratio average for  $K^+ K^-$  in the past editions of PDG from 1998 - 2006. PDG1998 quoted  $BR_{\chi_{c2} \rightarrow K^+ K^-} = (1.5 \pm 1.1) \times 10^{-3}$  [29] which was measured by DASP Collaboration [30]. In PDG2000 [31], it was averaged to be  $BR_{\chi_{c2} \rightarrow K^+ K^-} = (0.81 \pm 0.19) \times 10^{-3}$  with the measurement from BES Collaboration,  $(0.79 \pm 0.14 \pm 0.13) \times 10^{-3}$  [4]. In PDG2004 [32], this branching ratio was revised to be  $BR_{\chi_{c2} \rightarrow K^+ K^-} = (0.94 \pm 0.17 \pm 0.13) \times 10^{-3}$  by using  $BR(\psi(2S) \rightarrow \gamma \chi_{c2}) = (6.4 \pm 0.6)\%$  and  $BR(\psi(2S) \rightarrow J/\psi(1S) \pi^+ \pi^-) = 0.317 \pm 0.011$ . Then, in PDG2006 [28], this quantity was revised again to be  $BR_{\chi_{c2} \rightarrow K^+ K^-} = (0.77 \pm 0.14) \times 10^{-3}$ , but without explicit explanations. In contrast to this is that the branching ratio for  $K_s^0 \bar{K}_s^0$  has not experienced drastic changes. Further experimental investigation of these two channels will be necessary for understanding the  $\chi_{c2} \rightarrow PP$  decays.

### C. $\chi_{c0,2} \rightarrow SS$

The scalar pair production  $\chi_{c0} \rightarrow SS \rightarrow \pi^+ \pi^- K^+ K^-$  is analyzed at BES [6]. The intermediate  $K_0^* \bar{K}_0^*$  pair has a branching ratio of  $(1.05^{+0.39}_{-0.30}) \times 10^{-3}$  in its decay into  $\pi^+ \pi^- K^+ K^-$  and a set of  $f_0^i f_0^j$  pairs are measured, where  $i, j = 1, 2, 3$  denotes  $f_0(1710)$ ,  $f_0(1500)$  and  $f_0(1370)$ , respectively. The interesting feature is that the  $f_0(1370)f_0(1710)$  pair production is found to have the largest branching ratio in comparison with other  $f_0$  pairs. Theoretical interpretation for such an observation is needed and in Ref. [26], a parametrization for the SOZI and DOZI processes suggests that glueball- $q\bar{q}$  mixings can lead to an enhanced  $f_0(1370)f_0(1710)$  branching ratio in  $\chi_{c0}$  decays. However, due to the unavailability of the data for other scalar meson pair decays, estimate of the absolute branching ratios were not possible. Here, incorporated by the data for  $K_0^*(1430)\bar{K}_0^*(1430)$ , we expect to have more quantitative estimates of the  $\chi_{c0,2} \rightarrow SS$  branching ratios.

To proceed, several issues have to be addressed:

i) The scalars,  $f_0(1370)$ ,  $f_0(1500)$  and  $f_0(1710)$ , are assumed to be mixing states between scalar  $q\bar{q}$  and glueball  $G$ . On the flavor singlet basis, the state mixing can be expressed as

$$\begin{pmatrix} |f_0(1710)\rangle \\ |f_0(1500)\rangle \\ |f_0(1370)\rangle \end{pmatrix} = U \begin{pmatrix} |G\rangle \\ |s\bar{s}\rangle \\ |n\bar{n}\rangle \end{pmatrix} = \begin{pmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{pmatrix} \begin{pmatrix} |G\rangle \\ |s\bar{s}\rangle \\ |n\bar{n}\rangle \end{pmatrix}, \quad (10)$$

where  $x_i$ ,  $y_i$  and  $z_i$  are the mixing matrix elements determined by the perturbation transitions [33, 34, 35]. We adopt the mixing matrix  $U$  from Ref. [35]:

$$U = \begin{pmatrix} 0.36 & 0.93 & 0.09 \\ -0.84 & 0.35 & -0.41 \\ 0.40 & -0.07 & -0.91 \end{pmatrix}. \quad (11)$$

In order to examine the sensitivities of the branching ratios to the scalar meson structures in the numerical calculations, we will also apply several other mixing schemes [36, 37, 38] which are different from Ref. [35].

ii) In  $\chi_{c0,2} \rightarrow VV$  and  $PP$  the SU(3) flavor symmetry breaking turns to be at a magnitude of  $10 \sim 20\%$ . Namely, the deviation of the SU(3) flavor symmetry parameter  $R$  from unity is small. Due to lack of data we assume that a similar order of magnitude of the SU(3) flavor symmetry breaking appears in  $\chi_{c0} \rightarrow SS$ , and it is natural to assume  $R = 1$  as a leading order estimate.

We can thus determine the basic transition strength  $g_0$  via

$$\Gamma(\chi_{c0} \rightarrow K_0^* \bar{K}_0^*) = \frac{|\mathbf{p}| g_0^4 R^2 \mathcal{F}(|\mathbf{p}|)}{4\pi M_{\chi_{c0}}^2}, \quad (12)$$

where  $\mathbf{p}$  is the three-vector momentum of the final state  $K_0^*$  in the  $\chi_{c0}$ -rest frame, and  $\mathcal{F}(|\mathbf{p}|)$  is the form factor for the relative  $l$ -wave two-body decay. The partial decay width  $\Gamma(\chi_{c0} \rightarrow K_0^* \bar{K}_0^*)$  has been measured by BES [6]:

$$BR(\chi_{c0} \rightarrow K_0^* \bar{K}_0^* \rightarrow \pi^+ \pi^- K^+ K^-) = (10.44 \pm 1.57^{+3.05}_{-1.90}) \times 10^{-4}, \quad (13)$$

with  $BR(K_0^* \rightarrow K^+ \pi^-) = BR(\bar{K}_0^* \rightarrow K^- \pi^+) = 0.465$  [28].

iii) Since there is no constraint on the parameter  $t$ , we apply the flavor-blind assumption,  $t = 1$ , as a leading order approximation.

iv) In order to accommodate the BES data [6], we adopt the same branching ratios for  $f_0 \rightarrow PP$  as used in Ref. [35]:

$$BR(f_0(1710) \rightarrow \pi\pi) = 0.11 \times BR(f_0(1710) \rightarrow K\bar{K}) = 0.11 \times 0.6 , \quad (14)$$

$$BR(f_0(1500) \rightarrow \pi\pi) = 0.349 , \quad (15)$$

$$BR(f_0(1500) \rightarrow K\bar{K}) = 0.086 , \quad (16)$$

$$BR(f_0(1370) \rightarrow K\bar{K}) = 0.1 \times BR(f_0(1370) \rightarrow \pi\pi) = 0.1 \times 0.2 . \quad (17)$$

It should be noted that the final predictions for  $\chi_{c0} \rightarrow f_0^i f_0^j \rightarrow \pi^+ \pi^- K^+ K^-$  are sensitive to the above branching ratios. For the charged decay channel, factor 1/2 and 2/3 will be included in the branching ratio of  $f_0 \rightarrow K^+ K^-$  and  $\pi^+ \pi^-$ , respectively. Detailed analysis of the  $f_0$  states can be found in Ref. [39] and references therein.

Now, we are left with only one undetermined parameter  $r$ . By taking the measured branching ratio [6]:

$$BR(\chi_{c0} \rightarrow f_0(1370) f_0(1710)) \cdot BR(f_0(1370) \rightarrow \pi^+ \pi^-) \cdot BR(f_0(1710) \rightarrow K^+ K^-) = (7.12 \pm 1.46)^{+3.28}_{-1.68} \times 10^{-4} , \quad (18)$$

we determine  $r = 1.31 \pm 0.19$ . Consequently, predictions for other  $SS$  decay channels can be made and the results are listed in Table V.

A remarkable feature arising from the prediction is that  $BR(\chi_{c0} \rightarrow f_0(1370) f_0(1710))$  turns out to be the largest one in all the  $f_0$  pair productions with the constraint from  $K_0^*(1430) \bar{K}_0^*(1430)$ . As listed in Table V branching ratios of  $f_0(1370) f_0(1370)$  and  $f_0(1370) f_0(1500)$  are at order of 1%. Their signals in  $\pi^+ \pi^- K^+ K^-$  are suppressed due to their small branching ratios to  $\pi^+ \pi^-$  and  $K^+ K^-$  [40, 41, 42]. As a comparison decay channels with  $f_0(1710) \rightarrow K^+ K^-$  are less suppressed. Apart from the dominant channel  $f_0(1370) f_0(1710)$ , our calculation shows that  $\chi_{c0}$  has also large branching ratios into  $\pi^+ \pi^- K^+ K^-$  via  $f_0(1500) f_0(1710)$ . It shows that our results for  $\chi_{c0} \rightarrow f_0^i f_0^j \rightarrow \pi^+ \pi^- K^+ K^-$  provide a consistent interpretation for the BES data [6] though some of the predictions strongly depend on the estimates of the branching ratios of  $f_0 \rightarrow \pi^+ \pi^-$  and  $K^+ K^-$ .

The value of  $r = 1.31 \pm 0.19$  suggests an important contribution from the DOZI processes in  $\chi_{c0} \rightarrow f_0^i f_0^j$ , which is very different from the results in  $VV$  and  $PP$  channels. This certainly depends on the mixing matrix for the scalars, and also correlated with parameters  $R$  and  $t$ . At this moment, we still lack sufficient experimental information to constrain these parameters simultaneously. But it is worth noting that large contributions from the DOZI processes are also found in the interpretation [35] of the data for  $J/\psi \rightarrow \omega f_0(1710)$ ,  $\phi f_0(1710)$ ,  $\omega f_0(1370)$  and  $\phi f_0(1370)$  [41, 42]. The branching ratio for  $f_0(1710)$  recoiled by  $\omega$  in the  $J/\psi$  decays is found to be larger than it being recoiled by  $\phi$ , while branching ratio for  $f_0(1370)$  is larger than  $\omega f_0(1370)$ . Since  $f_0(1710)$  is coupled to  $K\bar{K}$  strongly and  $f_0(1370)$  prefers to couple to  $\pi\pi$  than  $K\bar{K}$ , a simple assumption for these two states is that  $f_0(1710)$  and  $f_0(1370)$  are dominated by  $s\bar{s}$  and  $n\bar{n}$ , respectively. Due to this, one would expect that their production via SOZI processes should be dominant, i.e.  $BR(J/\psi \rightarrow \phi f_0(1710)) > BR(J/\psi \rightarrow \omega f_0(1710))$  and  $BR(J/\psi \rightarrow \omega f_0(1370)) > BR(J/\psi \rightarrow \phi f_0(1370))$ . Surprisingly, the data do not favor such a prescription. In Ref. [35], we find that a glueball- $q\bar{q}$  mixing can explain the scalar meson decay pattern with a strong contribution from the DOZI processes. In fact, this should not be out of expectation if glueball- $q\bar{q}$  mixing occurs in the scalar sector.

We compute two additional decay channels for  $\chi_{c0} \rightarrow f_0^i f_0^j$ , i.e.  $\chi_{c0} \rightarrow f_0^i f_0^j \rightarrow \pi^+ \pi^- \pi^+ \pi^-$  and  $K^+ K^- K^+ K^-$ , which can be examined in experiment. The results are listed in the last two columns of Table V. It shows that the largest decay in the  $4\pi$  channel is via  $f_0(1370) f_0(1500)$ , and the smallest channel is via  $f_0(1500) f_0(1710)$ . Branching ratios are at order of  $10^{-4}$ , the same as the dominant  $f_0(1370) f_0(1500)$  channel. This means that an improved measurement will allow access to most of those intermediate states if the prescription is correct. In contrast, decays into four kaons are dominantly via  $f_0(1500) f_0(1710)$  and  $f_0(1370) f_0(1710)$  at order of  $10^{-5}$ , while all the others are significantly suppressed. The branching ratio pattern can, in principle, be examined by future experiment, e.g. at BESIII with much increased statistics. Nonetheless, uncertainties arising from the  $f_0 \rightarrow PP$  decays can be reduced.

It should be noted that our treatment for the SU(3) flavor symmetry breaking in order to reduce the number of free parameters can be checked by measuring  $\chi_{c0} \rightarrow a_0(1450) a_0(1450)$ . In the SU(3) symmetry limit, we predict  $BR_{\chi_{c0} \rightarrow a_0(1450) a_0(1450)} = 5.60 \times 10^{-3}$ , which is not independent of  $K_0^*(1430) \bar{K}_0^*(1430)$ . Experimental information about this channel will be extremely valuable for clarifying the role played by the DOZI processes.

In order to examine how this model depends on the scalar mixings, and learn more about the scalar meson structures, we apply another two mixing schemes from different approaches and compute the branching ratios for  $\chi_{c0} \rightarrow f_0^i f_0^j \rightarrow \pi^+ \pi^- K^+ K^-$ ,  $\pi^+ \pi^- \pi^+ \pi^-$  and  $K^+ K^- K^+ K^-$ . The first one is from Ref. [36] by Cheng *et al.* (Model-CCL) based on quenched lattice QCD calculations for the glueball spectrum, and the second one is from Ref. [37] by Giacosa *et al.* (Model-GGLF) in an effective chiral approach. We note that the mixing scheme of Ref. [38] with the truncated mixing matrix for the glueball and  $q\bar{q}$  part gives a similar result as Eq. (11).

In model-CCL, the mix matrix was given as

$$U = \begin{pmatrix} 0.859 & 0.302 & 0.413 \\ -0.128 & 0.908 & -0.399 \\ -0.495 & 0.290 & 0.819 \end{pmatrix}. \quad (19)$$

With the data from Eqs. (13) and (18), we determine  $r = 0.90 \pm 0.21$ . Predictions for other decay channels are given in Table VI.

In Model-GGLF, four mixing solutions were provided. We apply the first two as an illustration of the effects from the mixing schemes. The Solution-I gives

$$U = \begin{pmatrix} -0.06 & 0.97 & -0.24 \\ 0.89 & -0.06 & -0.45 \\ 0.45 & 0.24 & 0.86 \end{pmatrix}, \quad (20)$$

and Solution-II reads

$$U = \begin{pmatrix} -0.68 & 0.67 & -0.30 \\ 0.49 & 0.72 & -0.49 \\ 0.54 & 0.19 & 0.81 \end{pmatrix}. \quad (21)$$

We then determine  $r = 1.93 \pm 0.29$  and  $r = -2.07 \pm 0.79$  for Solution-I and II, respectively. The predictions for the branching ratios are listed in Tables VII and VIII.

Among all these outputs the most predominant feature is that large DOZI contributions are needed to explain the available data for  $\chi_{c0} \rightarrow f_0(1370)f_0(1710)$  and  $\chi_{c0} \rightarrow K_0^*(1430)\bar{K}_0^*(1430)$ . This also leads to the result that  $\chi_{c0} \rightarrow f_0(1370)f_0(1710) \rightarrow \pi^+\pi^-K^+K^-$  is a dominant decay channel. Thinking that all these scalar mixing schemes have quite different mixing matrix elements, the dominance of  $f_0(1370)f_0(1710)$  gives an impression that the  $SS$  branching ratios are not sensitive to the scalar wavefunctions. However, this is not the case, we note that the data cannot be explained if  $f_0(1710)$  is nearly pure glueball while  $f_0(1500)$  a pure  $s\bar{s}$ , namely, a mixing such as shown by the fourth solution of Ref. [37].

It turns more practical to extract information about the scalar structures in an overall study of the  $SS$  branching ratio pattern arising from  $\chi_{c0} \rightarrow SS \rightarrow \pi^+\pi^-K^+K^-$ ,  $4\pi$  and  $4K$ . For instance, in the  $\chi_{c0} \rightarrow SS \rightarrow 4K$ , the dominant channels are predicted to be via  $f_0(1370)f_0(1710)$  and  $f_0(1500)f_0(1710)$  in the mixing of Eq. (11), while in the other models the  $f_0(1500)f_0(1710)$  channel turns out to be small. In contrast, the  $f_0(1370)f_0(1370)$  channel is dominant in  $4\pi$  channel as predicted by Solution-II of Model-GGLF, while it is compatible with other channels in other solutions. Systematic analysis of these decay channels should be helpful for pinning down the glueball- $q\bar{q}$  mixings.

#### IV. SUMMARY

A systematic investigation of  $\chi_{c0,2} \rightarrow VV$ ,  $PP$  and  $SS$  in a general parametrization scheme is presented in line with the new data from BES and CLEO-c. It shows that the exclusive hadronic decays of the  $\chi_{c0,2}$  are rich of information about the roles played by the OZI-rule violations and SU(3) flavour breakings in the decay transitions. For  $\chi_{c0,2} \rightarrow VV$  and  $PP$ , we obtain an overall self-contained description of the experimental data. Contributions from the DOZI processes turn out to be suppressed. For the channels with better experimental measurement, i.e.  $\chi_{c0,2} \rightarrow VV$ , and  $\chi_{c0} \rightarrow PP$ , the SU(3) flavor symmetry is also better respected. Significant SU(3) breaking turns up in  $\chi_{c2} \rightarrow PP$  which is likely due to the poor status of the experimental data and future measurement at BESIII and CLEO-c will be crucial to disentangle this.

The BES data for  $\chi_{c0} \rightarrow SS$  allows us to make a quantitative analysis of the branching ratios in the scalar meson decay channel. In particular, it allows a test of the scalar  $f_0$  mixings motivated by the scalar glueball- $q\bar{q}$  mixing scenario. Including the new data for  $\chi_{c0} \rightarrow K_0^*\bar{K}_0^*$  from BES Collaboration, we find that the decay of  $\chi_{c0} \rightarrow f_0^if_0^j$  favors strong contributions from the DOZI processes. This phenomenon is consistent with what observed in  $J/\psi \rightarrow \phi f_0^i$  and  $\omega f_0^i$  [41, 42], where large contributions from the DOZI processes are also favored [35]. The  $SS$  decay branching ratio pattern turns out to be sensitive to the scalar mixing schemes. An overall study of  $\chi_{c0} \rightarrow SS \rightarrow \pi^+\pi^-K^+K^-$ ,  $4\pi$  and  $4K$  may be useful for us to gain some insights into the scalar meson structures and extract more information about the glueball signals in its production channel.

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Parameters	$\chi_{c0} \rightarrow VV$		$\chi_{c2} \rightarrow VV$	
	BES	PDG	BES	PDG
$r$	$0.203 \pm 0.192$	$0.176 \pm 0.197$	$-0.081 \pm 0.098$	$0.065 \pm 0.111$
$R$	$0.855 \pm 0.171$	$0.825 \pm 0.156$	$0.955 \pm 0.148$	$0.960 \pm 0.134$
$g_0$ (GeV $^{1/2}$ )	$0.291 \pm 0.038$	$0.297 \pm 0.042$	$0.371 \pm 0.039$	$0.348 \pm 0.034$

TABLE I: The parameters fitted for  $\chi_{c0,2} \rightarrow VV$  with data from BES [1, 2, 3] and the world averaged values from PDG.

Decay channel	$BR_{\chi_{c0} \rightarrow VV} (\times 10^{-3})$		$BR_{\chi_{c2} \rightarrow VV} (\times 10^{-3})$	
	BES	PDG	BES	PDG
$\phi\phi$	$1.0 (1.0 \pm 0.6)$	$0.9 (0.9 \pm 0.5)$	$2.0 (2.0 \pm 0.82)$	$1.9 (1.9 \pm 0.7)$
$\omega\omega$	$2.29 (2.29 \pm 0.71)$	$2.3 (2.3 \pm 0.7)$	$1.77 (1.77 \pm 0.59)$	$2.0 (2.0 \pm 0.7)$
$K^{*0}\bar{K}^{*0}$	$1.78 (1.78 \pm 0.48)$	$1.8 (1.8 \pm 0.6)$	$4.86 (4.86 \pm 1.04)$	$3.8 (3.8 \pm 0.8)$
$\rho\rho$	$3.457$	$3.755$	$7.532$	$5.816$
$\omega\phi$	$0.148$	$0.112$	$0.065$	$\sim 0$

TABLE II: The branching ratios obtained for  $\chi_{c0,2} \rightarrow VV$  by fitting the data from BES [1, 2, 3] and PDG average [28]. The data are listed in the bracket.

Parameters	$\chi_{c0} \rightarrow PP$	$\chi_{c2} \rightarrow PP$
$r$	$-0.120 \pm 0.044$	$-0.216 \pm 0.102$
$R$	$1.035 \pm 0.067$	$0.778 \pm 0.067$
$g_0$ (GeV $^{1/2}$ )	$0.366 \pm 0.007$	$0.283 \pm 0.008$

TABLE III: The parameters fitted for  $\chi_{c0,2} \rightarrow PP$  by combining the world-average data from PDG [28] and the newly published data from CLEO-c [7].

Decay channel	$BR_{\chi_{c0} \rightarrow PP} (\times 10^{-3})$		$BR_{\chi_{c2} \rightarrow PP} (\times 10^{-3})$	
	fit results	data	fit results	data
$\eta\eta$	2.51	$(1.9 \pm 0.5) [3.1 \pm 0.67]$	0.445	$[< 0.47]$
$\eta'\eta'$	1.68	$[1.7 \pm 0.46]$	0.076	$[< 0.31]$
$K^+K^-$	5.57	$(5.4 \pm 0.6)$	0.924	$(0.77 \pm 0.14)$
$K_s^0\bar{K}_s^0$	2.79	$(2.8 \pm 0.7)$	0.463	$(0.67 \pm 0.11)$
$\pi\pi$	7.25	$(7.2 \pm 0.6)$	2.123	$(2.14 \pm 0.25)$
$\eta\eta'$	0.089	$[< 0.50]$	0.095	$[< 0.23]$

TABLE IV: The branching ratios obtained for  $\chi_{c0,2} \rightarrow PP$  by fitting the world-average data from PDG (quoted in the round bracket) [28] together with the new data from CLEO-c (quoted in the square bracket) [7].

Decay channel	$BR(\chi_{c0} \rightarrow SS)(\times 10^{-3})$	$B_0 (\times 10^{-4})$	Exp. data ( $\times 10^{-4}$ )	$B_1 (\times 10^{-4})$	$B_2 (\times 10^{-5})$
$f_0(1370)f_0(1710)$	17.80	7.12	$(7.12 \pm 1.46)^{+3.28}_{-1.68}$	1.04	5.34
$f_0(1370)f_0(1370)$	13.14	0.17	< 2.9	2.33	0.13
$f_0(1370)f_0(1500)$	10.76	0.62	< 1.8	3.34	0.46
$f_0(1500)f_0(1370)$	10.76	0.25	< 1.4	3.34	0.46
$f_0(1500)f_0(1500)$	5.02	0.50	< 0.55	2.72	0.93
$f_0(1500)f_0(1710)$	6.18	4.31	< 0.73	0.63	7.98

TABLE V: The branching ratios obtained for  $BR_{\chi_{c0} \rightarrow SS}$ .  $B_0 \equiv BR(\chi_{c0} \rightarrow SS) \cdot BR(S \rightarrow \pi^+\pi^-) \cdot BR(S \rightarrow K^+K^-)$  are branching ratios to be compared with the BES data [6].  $B_1$  and  $B_2$  are branching ratios of  $\chi_{c0} \rightarrow SS \rightarrow \pi^+\pi^-\pi^+\pi^-$  and  $\chi_{c0} \rightarrow SS \rightarrow K^+K^-K^+K^-$ , respectively.

Decay channel	$BR(\chi_{c0} \rightarrow SS)(\times 10^{-3})$	$B_0 (\times 10^{-4})$	Exp. data ( $\times 10^{-4}$ )	$B_1 (\times 10^{-4})$	$B_2 (\times 10^{-5})$
$f_0(1370)f_0(1710)$	17.80	7.12	$(7.12 \pm 1.46)^{+3.28}_{-1.68}$	1.04	5.34
$f_0(1370)f_0(1370)$	5.06	0.07	< 2.9	0.90	0.05
$f_0(1370)f_0(1500)$	0.04	$\sim 0$	< 1.8	0.01	$\sim 0$
$f_0(1500)f_0(1370)$	0.04	$\sim 0$	< 1.4	0.01	$\sim 0$
$f_0(1500)f_0(1500)$	2.43	0.24	< 0.55	1.31	0.45
$f_0(1500)f_0(1710)$	0.74	0.52	< 0.73	0.08	0.96

TABLE VI: The branching ratios obtained for  $BR_{\chi_{c0} \rightarrow SS}$  in Model-CCL [36]. The notations are the same as Table V.

Decay channel	$BR(\chi_{c0} \rightarrow SS)(\times 10^{-3})$	$B_0 (\times 10^{-4})$	Exp. data ( $\times 10^{-4}$ )	$B_1 (\times 10^{-4})$	$B_2 (\times 10^{-5})$
$f_0(1370)f_0(1710)$	17.80	7.12	$(7.12 \pm 1.46)^{+3.28}_{-1.68}$	1.04	5.34
$f_0(1370)f_0(1370)$	97.15	1.29	< 2.9	17.27	0.97
$f_0(1370)f_0(1500)$	4.58	0.26	< 1.8	1.42	0.20
$f_0(1500)f_0(1370)$	4.58	0.11	< 1.4	1.42	0.20
$f_0(1500)f_0(1500)$	1.12	0.11	< 0.55	0.61	0.21
$f_0(1500)f_0(1710)$	0.22	0.15	< 0.73	0.22	0.28

TABLE VII: The branching ratios obtained for  $BR_{\chi_{c0} \rightarrow SS}$  with Solution-I of Model-GGLF [37]. The notations are the same as Table V.

Decay channel	$BR(\chi_{c0} \rightarrow SS)(\times 10^{-3})$	$B_0 (\times 10^{-4})$	Exp. data ( $\times 10^{-4}$ )	$B_1 (\times 10^{-4})$	$B_2 (\times 10^{-5})$
$f_0(1370)f_0(1710)$	17.80	7.12	$(7.12 \pm 1.46)^{+3.28}_{-1.68}$	1.04	5.34
$f_0(1370)f_0(1370)$	5.19	0.07	< 2.9	0.92	0.05
$f_0(1370)f_0(1500)$	2.09	0.12	< 1.8	0.65	0.09
$f_0(1500)f_0(1370)$	2.09	0.05	< 1.4	0.65	0.09
$f_0(1500)f_0(1500)$	2.45	0.24	< 0.55	1.33	0.45
$f_0(1500)f_0(1710)$	0.53	0.37	< 0.73	0.05	0.68

TABLE VIII: The branching ratios obtained for  $BR_{\chi_{c0} \rightarrow SS}$  with Solution-II of Model-GGLF [37]. The notations are the same as Table V.